Coalitions in Cooperative Wireless Networks

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***Abstract*— Cooperation between rational users in wireless networks is studied using coalitional game theory. Using the rate achieved by a user as its utility, it is shown that the stable coalition structure, i.e., set of coalitions from which users have no incentives to defect, depends on the manner in which the rate gains are apportioned among the cooperating users. Specifically, the stability of the *grand coalition* (GC), i.e., the coalition of all users, is studied. Transmitter and receiver cooperation in an interference channel (IC) are studied as illustrative cooperative models to determine the stable coalitions for both flexible (*transferable*) and fixed (*non-transferable*) apportioning schemes. It is shown that the stable sum-rate optimal coalition when only receivers cooperate by jointly decoding (transferable) is the GC. The stability of the GC depends on the detector when receivers cooperate using linear multiuser detectors (non-transferable). Transmitter cooperation is studied assuming that all receivers cooperate perfectly and that users outside a coalition act as jammers. The stability of the GC is studied for both the case of perfectly cooperating transmitters (transferrable) and under a *partial decode-and-forward* strategy (non-transferable). In both cases, the stability is shown to depend on the channel gains and the transmitter jamming strengths.**

***Index Terms*— Coalitional games, cooperative communications, interference channel.**

I. Introduction

Cooperation in wireless networks results when nodes exploit the broadcast nature of the wireless medium and use their power and bandwidth resources to mutually enhance transmissions (see, for e.g., [1], [2], [3] and the references therein). In general, it is assumed that all the network nodes are willing to cooperate. However, when rational (self-interested) users are allowed to cooperate it is necessary to examine whether the cooperation of all users, i.e., the *grand coalition* (GC) of all users, can be taken for granted. In fact, cooperation may involve significant costs and the greatest immediate benefits may not be achieved by the users that bear the greatest immediate cost. An additional disincentive to cooperation may result from the rules by which the cooperative gains are distributed among participating users. In fact, for maximum gains users may prefer to cooperate with a select set of users to form *coalitions* that are closed to cooperation from users outside the group. For example, consider a multi-user wireless network where users labeled A, B, and C are decoded at a central receiver. Cooperating users share the benefit of having their signals jointly decoded at the receiver while a user that chooses not to cooperate is decoded independently and is subject to interference from the other users.

One can verify that the multiaccess channel (MAC) that results when all three users cooperate achieves the maximum information-theoretic three-user sum-rate [4, 14.3]. However, it is not clear if the GC is also a *stable* coalition, i.e., a coalition whose users do not have an incentive to leave (for larger rates). For example, consider an apportionment strategy where the sum-rate achieved is divided equally among the users in a coalition. In Fig. 1 we demonstrate the stability of the various coalitions as a function of the received signal-to-noise ratio (SNR) of each user. Observe that the grand coalition is desirable only when all users have similar SNR values. Further, for arbitrary SNR values, the users in the stable coalitions benefit from the exclusion of the weak interferer. Thus, even in this relatively simple example we see that user cooperation is desirable only when the aggregate benefits of cooperation provide adequate incentives to all participating users.

We use the framework of coalitional game theory to determine the stable coalition structure, i.e., a set of coalitions whose users do not have incentives to break away, when wireless nodes are allowed to cooperate (see for e.g., [5], [6]). We consider a K-link interference channel (IC) [7] as an illustrative network model to determine the stable coalitions when transmitters or receivers are allowed to cooperate. Specifically, we focus on the stability of the grand coalition and seek to understand if the GC also maximizes the utilities of all the users. For specific encoding and decoding schemes, we model the maximum achievable information-theoretic rate as a measure of a user's utility. The encoding and decoding schemes also determine the manner in which the rate gains can be apportioned between the cooperating users in a coalition. Coalitional games are classified into two types based on the apportioning of gains among users in a coalition [8, Section IV]: i) a *transferable utility* (TU) game where the total rate achieved is apportioned arbitrarily between the users in a coalition subject to feasibility constraints and ii) a *non-transferable utility* (NTU) game where the apportioning strategies have additional constraints that prevent arbitrary apportioning.

In [9], [10], we apply results from information theory and TU games to study the stable coalition structure when only receivers in an IC cooperate by jointly decoding their received signals. We show that the GC of receivers is the stable sum-

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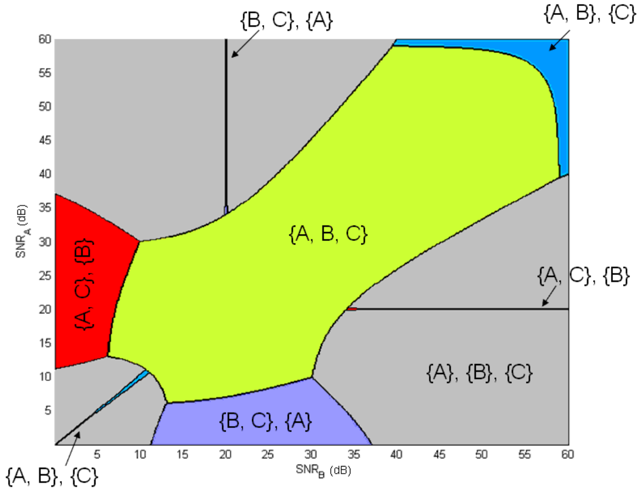

Fig. 1. Stable Coalition Structures as a function of the SNR values of users A and B and $SNR_C = 20$ dB.

rate maximizing coalition structure. On the other hand, for the case where the receivers cooperate using linear multiuser detectors, we show that the GC is always the stable coalition for the MMSE detector and is stable only in the high signal-to-noise ratio (SNR) regime for the decorrelating detector. We briefly review our results in Section IV.

In this paper, we study the formation of stable coalitions when transmitters are allowed to cooperate in a K-link IC. The cooperative strategies and rate regions for a 2-link IC with varying degrees of transmitter and receiver cooperation is studied in [11], [12] and the references therein. For a K-link IC, there is a combinatorial explosion in the ways in which the transmitters can cooperate. Thus, knowledge of the stable coalition structures can be useful in choosing the appropriate cooperative strategies. We assume that the K receivers jointly decode their received signals thus simplifying the IC to a multi-access (MAC) channel with a multi-antenna receiver. We also assume that transmitters in a coalition have no knowledge of the transmission strategies of the users outside. We model the lack of transmit information between competing coalitions as a *jamming* game, i.e., we assume that each coalition determines its stability by assuming worst case jamming interference from other coalitions. We first study the TU game that results when the transmitters in a coalition cooperate perfectly, i.e., each transmitter has perfect knowledge of the messages of the other transmitters in its coalition. We prove that the game is *cohesive* [8, chap. 13], i.e., the largest K-user sum-rate is achieved by the GC. This allows us to show that the GC is the only viable stable coalition structure [8, p. 258], i.e., no stable coalition structure exists when the GC is not stable. Finally, using examples we demonstrate that the GC is not always stable and that the stability depends on the relative strengths of the user channels to the destination.

We also study the NTU game that results when all the transmitters in a coalition decode and jointly forward a part of their message streams via a *partial decode-and-forward* (PDF) strategy [13], [14]. We assume perfectly cooperating co-located receivers with fixed channel gains thus simplifying the IC to a cooperative MAC. Motivated by the results for the perfect transmitter cooperation game, we focus on a class of channels where all the users are *clustered*, i.e., their inter-user links are stronger than the links between the users and the destination. For this class, we prove that the achievable rate region is maximized when transmitters in a coalition decode all messages from one another thus generalizing the results for a two-user cooperative MAC in [15, Proposition 1]. However, using examples, we show that when the jamming is weak, users may have incentives to break away from the cluster, i.e., the game may not be cohesive. These results for clustered users also point to the fact that for the general class of channels with arbitrary inter-user links the game may not be cohesive in general.

This paper is organized as follows. In Section II we provide an overview of coalitional game theory. In section III we introduce the system models. In Section IV we review our results on receiver cooperation. In Section V, we study transmitter cooperation as a coalitional game using two different cooperation models. We conclude in Section VI.

II. Coalitional Game Theory for Receiver and Transmitter Cooperation

We use the framework of coalitional game theory to determine the stable rate maximizing cooperative coalitions in a wireless network. To determine stability one must in general take into account the fact that the rate achieved by a coalition is also affected by the actions of the users outside the coalition. However, determining the stable coalition structures for such a general model is not straightforward [8, p. 258]. Thus, it is common practice to assume that a game is in *characteristic function form* (CFF), i.e., the utilities achieved by the users in a coalition are unaffected by those outside it [16].

When only receivers cooperate, the game is in CFF. This is due to the fact that the transmitters in these models do not cooperate. In fact, for a fixed encoding at the transmitters, the rate achieved by any coalition only depends on the combined interference presented by the users outside the coalition and not on the coalition structures to which they belong. On the other hand, the games resulting from both kinds of transmitter cooperation models are not in CFF because the cooperative strategies of users outside a coalition affects the rates achieved by the members of a coalition. We convert the game to a CFF by considering a *jamming game*, i.e., we assume that a coalition assumes that the users outside cooperate to act as worst case jammers.

Games in CFF can be further categorized as TU and NTU games depending on whether the cooperative gains are divided arbitrarily or in a constrained manner, respectively. We define both games and their properties below.

Definition 1: A coalitional game with transferable utility $\langle\mathcal{K}, v\rangle$ is defined as [8, Chap. 13]

- a finite set of users $\mathcal{K}$,
- a value $v(\mathcal{S}) \in \mathbb{R}_+$ for all $\mathcal{S} \subseteq \mathcal{K}$ with $v(\{\phi\}) = 0$.

A coalition structure is a partition of the set $\mathcal{K}$, and thus the number of coalition structures, i.e., the number of possible partitions of $\mathcal{K}$, grows exponentially with K [17]. In fact, it

has been shown that finding the sum-rate maximizing coalition structure is an NP-complete problem [17]. To this end, the following properties of a TU game greatly simplify such a search.

Definition 2: A coalitional game with transferable utility is said to be *cohesive* if the value of the grand coalition formed by the set of all users $\mathcal{K}$ is at least as large as the sum of the values of any partition of $\mathcal{K}$, i.e.

$$\sum_{n=1}^{N} v(\mathcal{S}_n) \leq v(\mathcal{K}) \tag{1}$$

for any partition $(\mathcal{S}_1, \ldots, \mathcal{S}_N)$ of $\mathcal{K}$ where $2 \leq N \leq K$.

Remark 3: A TU game that is cohesive has the GC as the optimal coalition structure [8, p. 258], i.e., the sum of the utilities of all the users is maximum. This follows from the fact that all other coalition structures will be unstable as every user has an incentive to join the GC and benefit from a redistribution of total utility.

In addition to being cohesive, a TU coalitional game can also be superadditive which is defined as follows.

Definition 4: A coalitional game with transferable payoff is said to be superadditive if for any two disjoint coalitions $\mathcal{S}_1$ and $\mathcal{S}_2$, we have

$$v(\mathcal{S}_1 \cup \mathcal{S}_2) \geq v(\mathcal{S}_1) + v(\mathcal{S}_2). \tag{2}$$

Remark 5: Comparing (1) and (2), we see that superadditivity requires the cohesive property to hold for any two disjoint subsets of $\mathcal{K}$ with respect to their union.

We refer to a vector describing the share of the rate (payoffs) received by the members (players) of a coalition as a *payoff vector*.

Definition 6: For any coalition $\mathcal{S}$, a vector $\underline{x}_{\mathcal{S}} = (x_m)_{m\in\mathcal{S}}$ of real numbers is a *$\mathcal{S}$-feasible payoff vector* if $x(\mathcal{S}) = \sum_{m\in\mathcal{S}} x_m = v(\mathcal{S})$. The $\mathcal{K}$-feasible payoff vector is referred to as a *feasible payoff profile*.

Of all possible coalition structures, the ones that are stable are of most interest. Further, due to the complexity of finding stable coalition structures for non-cohesive games where the GC does not achieve the largest value, coalitional games that are cohesive are the easiest to study. For wireless networks, such games also optimize the spectrum utilization. In the following definition, we assume that the game is cohesive and thus the GC is the only possible stable coalition.

Definition 7: The *core*, $C(v)$, of a coalitional game with transferable payoff, $\langle\mathcal{K}, v\rangle$, is the set of feasible payoff profiles $\underline{x}_{\mathcal{K}}$ for which there is no coalition $\mathcal{S} \subset \mathcal{K}$ and a corresponding $\mathcal{S}$-feasible payoff vector $\underline{y}_{\mathcal{S}} = (y_m)_{m\in\mathcal{S}}$ such that $y_m > x_m$ for all $m \in \mathcal{S}$.

For TU games, Definition 7 simplifies to the condition that the feasible payoff profiles $\underline{x}_{\mathcal{K}}$ in the core satisfy

$$x(\mathcal{S}) = \textstyle\sum_{m\in\mathcal{S}} x_m \geq v(\mathcal{S}) \quad \text{for all } \mathcal{S} \subset \mathcal{K} \tag{3}$$

$$x(\mathcal{K}) = \textstyle\sum_{m\in\mathcal{K}} x_m = v(\mathcal{K}). \tag{4}$$

This follows from the fact that in a game with transferable payoff if there exists a coalition $\mathcal{S}$ with $v(\mathcal{S}) > x(\mathcal{S})$ then we can always find a $\mathcal{S}$-feasible payoff vector $\underline{y}_{\mathcal{S}}$ such that $y_k > x_k$, for all $k \in \mathcal{S}$. Such an assignment can result, for instance, when the $\mathcal{S}$-feasible payoff vector $\underline{y}_{\mathcal{S}}$ is constructed by assigning to each link $k \in \mathcal{S}$, the payoff $\overline{x}_k$ and then uniformly apportioning the surplus payoff $v(\mathcal{S}) - x(\mathcal{S})$ between links in $\mathcal{S}$. We use this equivalent definition to determine the stability of the core. Finally, we remark that determining the non-emptiness of the core simplifies to determining whether the linear program defined by the inequalities in (3) and (4) is feasible.

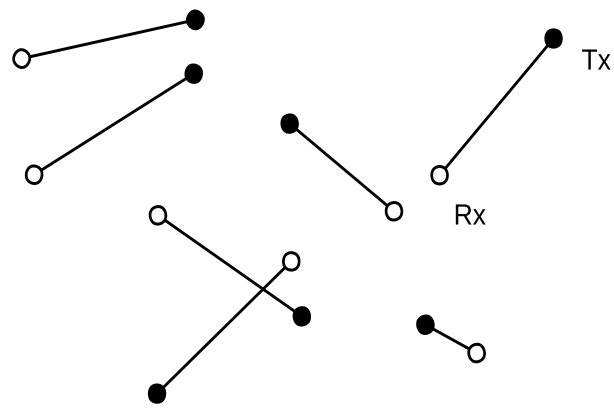

Fig. 2. An interference channel with K transmit-receive links.

We formally define an NTU game and its properties below [8, p. 268].

Definition 8: A coalitional game with non-transferable utility $\langle\mathcal{K}, \mathcal{V}\rangle$ consists of

- A finite set $\mathcal{K}$ of K players,
- A set function $\mathcal{V} : \mathcal{S} \rightarrow \mathbb{R}_+^K$ such that for all $\mathcal{S} \subseteq \mathcal{K}$
  - $\mathcal{V}(\phi) = \phi$ (normalized)
  - $\mathcal{V}(\mathcal{S})$ is a non-empty closed subset of $\mathbb{R}_+^K$ such that the components of the rate tuples in $\mathcal{V}(\mathcal{S})$ whose indices correspond to players not in $\mathcal{S}$ can be arbitrary,
  - for any length-K vectors $\underline{x} \in \mathcal{V}(\mathcal{K})$ and $\underline{y} \in \mathbb{R}_+^K$ with entries $y_k \leq x_k$, for all k, we have $\underline{y} \in \mathcal{V}(\mathcal{K})$ (comprehensive).

Definition 9: An NTU coalitional game $\langle\mathcal{K}, \mathcal{V}\rangle$ is cohesive if and only if

$$\bigcap_{n=1}^{N} \mathcal{V}(\mathcal{S}_n) \subseteq \mathcal{V}(\mathcal{K}) \tag{5}$$

where $\{\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_N\}$ is any partition of $\mathcal{K}$ where $2 \leq N \leq K$.

As with TU games, we focus on the stability of the GC and define a core of a NTU game that is cohesive.

Definition 10: The core $\mathcal{C}(\mathcal{K}, \mathcal{V})$ of an NTU coalitional game $\langle\mathcal{K}, \mathcal{V}\rangle$ is the set of payoff vectors $\underline{x} \in \mathcal{V}(\mathcal{K})$ such that there is no coalition $\mathcal{S}$ and payoff vector $\underline{y} \in \mathcal{V}(\mathcal{S})$ such that $y_k > x_k$ for all $k \in \mathcal{S}$.

III. Channel and Cooperation Models

A. Channel Model

Our network consists of K transmitter-receiver pairs (links), indexed by the set $\mathcal{K} = \{1, \ldots, K\}$ [7] (see Fig. 2). We model

each link as an additive white Gaussian noise channel with fixed channel gains. The received signal at receiver m is given by

$$Y_m = \sum_{k=1}^{K} \sqrt{h_{m,k}} X_k + Z_m \quad m \in \mathcal{K} \tag{6}$$

where $h_{m,k}^{1/2}$ is the channel gain between transmitter k and receiver m. The noise entries $Z_m \sim \mathcal{CN}(0,1)$, for all m, are independent, identically distributed (i.i.d), proper complex zero-mean unit-variance Gaussian random variables. The transmit power at transmitter k is constrained as

$$E|X_k|^2 \leq P_k \quad \text{for all } k \in \mathcal{K}. \tag{7}$$

We assume that the transmitters employ Gaussian signaling subject to (7). For the case where the receivers are co-located, our model simplifies to a MAC where all the transmitters communicate with the same destination, denoted as d such that $Y_d = Y_k$ for all k. Finally, we write $X_{\mathcal{S}} = \{X_k : k \in \mathcal{S}\}$ for all $\mathcal{S} \subseteq \mathcal{K}$ and $\mathcal{S}^c$ as the complement of $\mathcal{S}$ in $\mathcal{K}$. Finally, throughout the paper, we use the words user and transmitter interchangeably.

B. Cooperation Models

a) Receiver cooperation via Joint decoding: We assume that the receivers that cooperate communicate via noise-free links and that the transmitters do not cooperate. We assume that a coalition of cooperating receivers treats signals from transmitters outside the coalition as interference. For the channel in (6), each non-singleton coalition can thus be modeled as a single-input, multiple-output Gaussian multiple access channel (SIMO-MAC) whose capacity region is known [18] and achieved by the Gaussian input signaling chosen.

b) Receiver cooperation using Linear multiuser detectors: We assume an IC with co-located receivers thereby simplifying the channel to a single-antenna MAC. We consider a BPSK modulated, synchronized CDMA system with no power control such that the correlation between any two user signature sequences is ρ. We write the signal at the receiver as [19, p. 19]

$$y(t) = \sum_{k=1}^{K} \sqrt{P} h_k b_k s_k(t) + \sigma n(t), \quad t \in [0,T] \tag{8}$$

where P is the common transmit power of all users, h_k is the channel gain from user k to the receiver, $b_k \in \{+1,-1\}$ is the bit transmitted by user k in the bit interval $[0,T]$, $s(t)$ is the signature sequence of user k, and $n(t)$ is an additive white Gaussian noise process with unit variance. The received signal is filtered through a bank of K matched filters to obtain a $K \times 1$ received signal vector [19]

$$\mathbf{y} = \mathbf{RAb} + \mathbf{n} \tag{9}$$

where $\mathbf{R} \in \mathcal{R}^{K \times K}$ is a signature sequence cross correlation matrix, $\mathbf{A}$ is a diagonal matrix containing the received amplitudes $\sqrt{P} h_k$, for all k, $\mathbf{b}$ is an $K \times 1$ vector of transmitted bits, and $\mathbf{n}$ is a Gaussian random vector with zero mean and covariance $\sigma^2 \mathbf{R}$.

Transmitter Cooperation: We study two models for transmitter cooperation in a K-link IC. In both cases, we assume that the receivers of all the links jointly decode (see Fig. 5). Further, for simplicity, under PDF, we assume co-located receivers thus simplifying the IC to a cooperative MAC. Finally, in both cases, we assume that each coalition is affected by worst-case jamming by competing coalitions.

c) Perfect cooperation: For perfect transmitter cooperation each non-singleton coalition can be modeled as a multi-input, K-output MIMO channel with per-antenna power constraints. The transmitters in a coalition maximize their MIMO sum-capacity [18] subject to worst case jamming from other coalitions.

d) Partial decode-and-forward: We consider a MAC where a coalition of transmitters cooperate via a PDF scheme [13], [1], [14]. We assume full duplex communications at the cooperating transmitters. The received signals Y_d and Y_j at the destination and at user j, respectively, are

$$Y_d = \sum_{k=1}^{K} \sqrt{h_{d,k}} X_k + Z_d \tag{10}$$

$$Y_j = \sum_{k \in \mathcal{K}, k \neq j} \sqrt{h_{j,k}} X_k + Z_j \quad \text{for all } j \in \mathcal{K}. \tag{11}$$

where $h_{j,k}^{1/2}$ is the channels gain from user k to user j, and Z_d and Z_j are zero-mean unit variance proper complex Gaussian noise variables. We focus on a class of *clustered* channels, i.e., a network where

$$h_{m,k} > h_{d,k} \quad \text{for all } m \in \mathcal{K}, m \neq k. \tag{12}$$

This represents a model where the users are most likely to cooperate to overcome a relatively poor direct channel to the destination.

IV. Receiver Cooperation

In [9], [10], we determine the stable coalitions when receivers cooperate in an IC. The cooperation models are described in Section III-B and we present the results here.

A. Receiver Cooperation via Joint Decoding (TU game)

Consider the TU game that results when cooperating receivers in a K-link IC jointly decode their received signals (Fig. 3). For fixed channel gains, we define the value $v(\mathcal{S})$ of a coalition $\mathcal{S}$ of links as the maximum information-theoretic sum-rate achieved by the links in $\mathcal{S}$, i.e., [9]

$$v(\mathcal{S}) = \max_{\underline{R}_{\mathcal{S}} \in \mathcal{C}_{\mathcal{S}}} \sum_{i \in \mathcal{S}} R_i = \max_{P_{X_{\mathcal{S}}}} I(X_{\mathcal{S}}; Y_{\mathcal{S}}) \tag{13}$$

where $\underline{R}_{\mathcal{S}} = (R_i)_{i \in \mathcal{S}}$ is the vector of rates for links in $\mathcal{S}$ and $\mathcal{C}_{\mathcal{S}}$ is the capacity region of the SIMO-MAC formed by the links in $\mathcal{S}$. For the white Gaussian channel considered, the input distribution $P_{X_{\mathcal{S}}}$ maximizing (13) is zero-mean independent Gaussian signaling at each transmitter in $\mathcal{S}$ with variance set to the maximum transmit power in (7). The value $v(\mathcal{S})$ of a coalition $\mathcal{S}$ can be apportioned between its members in any arbitrary manner. Depending on its allocated share of $v(\mathcal{S})$, a receiver may decide to break away from the coalition $\mathcal{S}$ and join another coalition where it achieves a greater rate. For this model, we prove the following results (see [9]).

Theorem 11: The grand coalition maximizes spectrum utilization in the joint decoding receiver cooperation coalitional game.

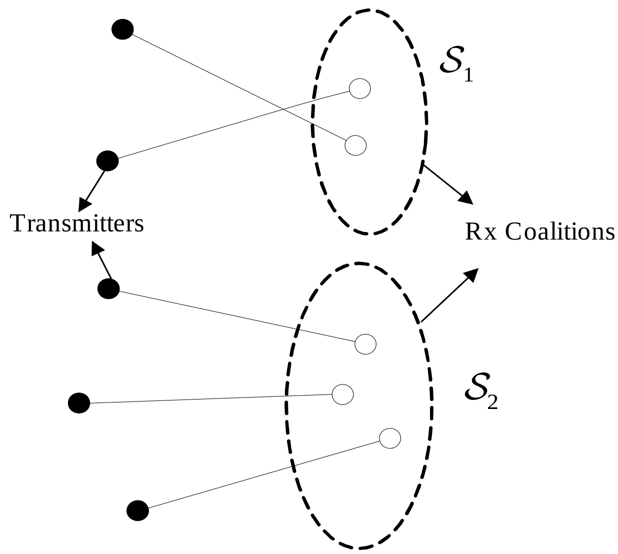

Fig. 3. Receiver coalitions formed in a K-link IC when receivers cooperate via joint decoding and transmitters do not cooperate.

Proof: From definition 4 for a superadditive game, the sum-rate of all links is maximized by the grand coalition. Since maximizing the sum-rate is equivalent to maximizing the utilization of the shared spectrum, we only need to show that the value of a coalition for this receiver cooperation coalitional game is a superadditive function.

Consider two coalitions $\mathcal{S}_1$ and $\mathcal{S}_2$ such that $\mathcal{S}_1 \cap \mathcal{S}_2 = \phi$. In order to prove that $v(\mathcal{S})$ is superadditive, we need to show that

$$I(X_{\mathcal{S}_1 \cup \mathcal{S}_2}; Y_{\mathcal{S}_1 \cup \mathcal{S}_2}) \geq I(X_{\mathcal{S}_1}; Y_{\mathcal{S}_1}) + I(X_{\mathcal{S}_2}; Y_{\mathcal{S}_2}) \tag{14}$$

We expand $I(X_{\mathcal{S}_1 \cup \mathcal{S}_2}; Y_{\mathcal{S}_1 \cup \mathcal{S}_2})$ as

$$\begin{aligned} I(X_{\mathcal{S}_1 \cup \mathcal{S}_2}; Y_{\mathcal{S}_1 \cup \mathcal{S}_2}) &= I(X_{\mathcal{S}_1}; Y_{\mathcal{S}_1}) + I(X_{\mathcal{S}_1}; Y_{\mathcal{S}_2} | Y_{\mathcal{S}_1}) \\ &\quad + I(X_{\mathcal{S}_2}; Y_{\mathcal{S}_2} | X_{\mathcal{S}_1}) + I(X_{\mathcal{S}_2}; Y_{\mathcal{S}_1} | Y_{\mathcal{S}_2}, X_{\mathcal{S}_1}) \end{aligned} \tag{15}$$

Further expanding $I(X_{\mathcal{S}_2}; Y_{\mathcal{S}_2} | X_{\mathcal{S}_1})$, we have

$$I(X_{\mathcal{S}_2}; Y_{\mathcal{S}_2} | X_{\mathcal{S}_1}) = H(X_{\mathcal{S}_2}) - H(X_{\mathcal{S}_2} | Y_{\mathcal{S}_2}, X_{\mathcal{S}_1}) \tag{16}$$

$$\geq I(X_{\mathcal{S}_2}; Y_{\mathcal{S}_2}) \tag{17}$$

where (16) follows from the independence of the transmitter signals and the inequality in (17) from the fact that conditioning reduces entropy. Finally, comparing (15) with (14) and using the fact that mutual information is non-negative, we have that the joint decoding receiver cooperation coalitional game is superadditive. ■

Theorem 12: The GC is the stable coalition structure that maximizes the spectrum utilization in the interference channel with jointly decoding cooperating receivers.

Proof: Since the interference channel coalitional game is superadditive, we need only consider the definition of the core in the context of the grand coalition. Any feasible payoff profile $\underline{R}_{\mathcal{K}} = (R_k)_{k \in \mathcal{K}}$ that lies in the capacity region, $\mathcal{C}_{\mathcal{K}}$, of a SIMO-MAC with K independent transmitters and a K-antenna receiver satisfies the inequalities

$$\sum_{k \in \mathcal{S}} R_k \leq I(X_{\mathcal{S}}; Y_{\mathcal{K}} | X_{\mathcal{S}^c}) \quad \forall \mathcal{S} \subseteq \mathcal{K}. \tag{18}$$

For Gaussian MIMO-MAC channels, the bounds in (18) are maximized by independent Gaussian signaling at the transmitters. We claim that every feasible payoff profile $\underline{R}_{\mathcal{K}}$ on the dominant face of the capacity region $\mathcal{C}_{\mathcal{K}}$ lies in the core. By the equivalent definition of the core, in order to prove that a $\underline{R}_{\mathcal{K}}$ satisfying (18) lies in the core, we need to show that

$$\sum_{k \in \mathcal{S}} R_k \geq v(\mathcal{S}) \quad \forall\ \mathcal{S} \subseteq \mathcal{K} \tag{19}$$

Since $\underline{R}_{\mathcal{K}}$ is a feasible payoff profile, i.e., $\sum_{k \in \mathcal{K}} R_k = v(\mathcal{K})$, we have

$$\sum_{k \in \mathcal{K}} R_k = \sum_{k \in \mathcal{S}} R_k + \sum_{k \in \mathcal{S}^c} R_k = I(X_{\mathcal{K}}; Y_{\mathcal{K}}). \tag{20}$$

We rewrite (20) above as

$$\sum_{k \in \mathcal{S}} R_k = I(X_{\mathcal{K}}; Y_{\mathcal{K}}) - \sum_{k \in \mathcal{S}^c} R_k \tag{21}$$

$$\geq I(X_{\mathcal{S}}, X_{\mathcal{S}^c}; Y_{\mathcal{K}}) - I(X_{\mathcal{S}^c}; Y_{\mathcal{K}} | X_{\mathcal{S}}) \tag{22}$$

$$= I(X_{\mathcal{S}}; Y_{\mathcal{S}}, Y_{\mathcal{S}^c}) \tag{23}$$

$$= I(X_{\mathcal{S}}; Y_{\mathcal{S}}) + I(X_{\mathcal{S}}; Y_{\mathcal{S}^c} | Y_{\mathcal{S}}) \tag{24}$$

$$\geq I(X_{\mathcal{S}}; Y_{\mathcal{S}}) \tag{25}$$

where the inequality in (22) follows from (18) assuming optimal Gaussian signaling at the transmitters, (23) follows from applying the chain rule for mutual information in (22), and (25) follows from the non-negativity of mutual information. Thus, we have

$$\sum_{k \in \mathcal{S}} R_k \geq I(X_{\mathcal{S}}; Y_{\mathcal{S}}) = v(\mathcal{S}) \tag{26}$$

The above inequality implies that every point on the *dominant face of* $\mathcal{C}_{\mathcal{S}}$, i.e., on the plane that maximizes the sum rate of all transmitters, corresponds to a feasible rate payoff profile that lies in the core. Thus, the core for the interference channel coalitional game is not only non-empty but is, in general, also non-unique. ■

B. Receiver Cooperation using Multiuser Detectors (NTU game)

In [10], we study the stability of the coalitional game that results when the co-located receivers in an IC use a linear multiuser detector (MUD) to cooperatively process their matched filter signals [19, Chaps. 5, 6]. As described in Section III-B, the transmitters use random signature sequences to transmit binary signals. We consider a decorrelating [19, Chap. 5] and a MMSE detector [19, Chap. 6] and in both cases determine the SNR regimes for which the GC is the stable sum-rate maximizing coalition structure. An example of a coalition of multiuser detectors in shown in Fig. 4.

For any coalition $\mathcal{S} \subset \mathcal{K}$, the received signal vector for this coalition is given by

$$\mathbf{y}_{\mathcal{S}} = \mathbf{R}_{\mathcal{S}} \mathbf{A}_{\mathcal{S}} \mathbf{b}_{\mathcal{S}} + \mathbf{R}_{\mathcal{S}^c} \mathbf{A}_{\mathcal{S}^c} \mathbf{b}_{\mathcal{S}^c} + \mathbf{n}_{\mathcal{S}} \tag{27}$$

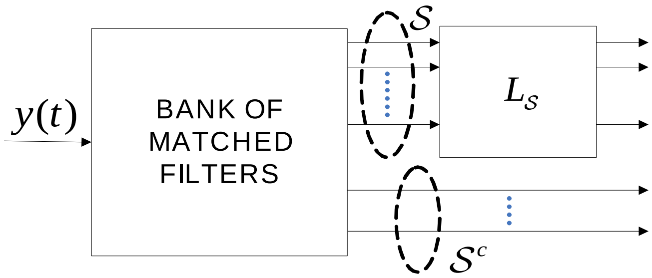

Fig. 4. Coalition of links for a decorrelating detector coalitional game.

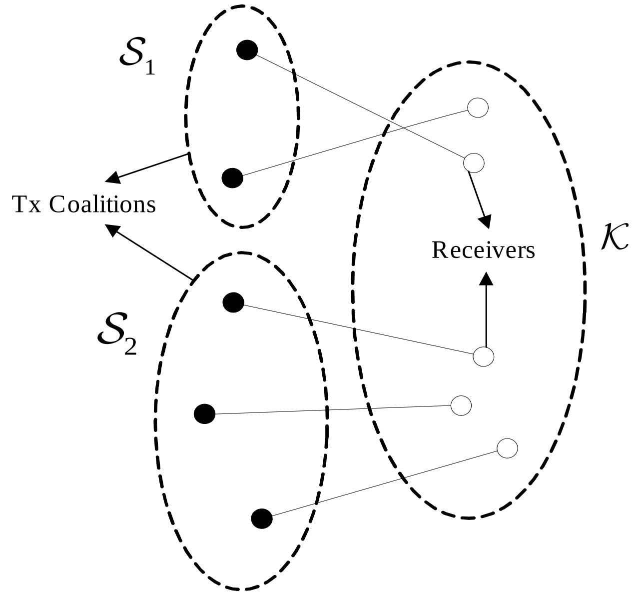

Fig. 5. Transmitter coalitions in a K-link IC when transmitters cooperate via noise-free links and all K receivers cooperate.

where $\mathbf{R}_{\mathcal{S}}$ is the cross correlation matrix of the transmit signature sequences in $\mathcal{S} \subseteq \mathcal{K}$, $\mathbf{A}_{\mathcal{S}}$ is a diagonal matrix containing the received amplitudes $\sqrt{P}h_k$ for all $k \in \mathcal{S}$, $\mathbf{b}_{\mathcal{S}}$ is the vector of bits from transmitters in $\mathcal{S}$, and $\mathbf{n}_{\mathcal{S}}$ is a random Gaussian vector with zero mean and covariance matrix $\sigma^2\mathbf{R}_{\mathcal{S}}$. The $|\mathcal{S}| \times |\mathcal{S}^c|$ matrix $\mathbf{R}_{\mathcal{S}^c}$ contains the cross correlations between the signature sequences of users in $\mathcal{S}$ and $\mathcal{S}^c$, i.e., $(\mathbf{R}_{\mathcal{S}^c})_{ij} = \rho$, for all $i = 1, 2, \ldots, |\mathcal{S}|$ and $j = 1, 2, \ldots, K - |\mathcal{S}|$. The $|\mathcal{S}^c| \times |\mathcal{S}^c|$ diagonal matrix $\mathbf{A}_{\mathcal{S}^c}$ and the $|\mathcal{S}^c|$-length vector $\mathbf{b}_{\mathcal{S}^c}$ contains the amplitudes and bits, respectively, of transmitters in $\mathcal{S}^c$.

A multiuser detector for the coalition $\mathcal{S}$ applies a linear transformation $\mathbf{L}_{\mathcal{S}}$ and the resulting vector $\mathbf{L}_{\mathcal{S}}\mathbf{y}_{\mathcal{S}}$ is used to decode the bits from the transmitters in $\mathcal{S}$. For the decorrelating receiver, $\mathbf{L}_{\mathcal{S}} = \mathbf{R}_{\mathcal{S}}^{-1}$ and for the MMSE receiver, $\mathbf{L}_{\mathcal{S}} = \left(\mathbf{R}_{\mathcal{S}} + \sigma^2\mathbf{A}_{\mathcal{S}}^{-2}\right)^{-1}$. Links within a coalition benefit from interference suppression offered by their MUD. The coalitional games for both detectors are NTU games since linear MUDs achieve a specific rate tuple for each user in the coalition. Finally, for both detectors we assume that the rate achieved by each link is a monotonically increasing function of its signal-to-interference noise ratio (SINR) at the receiver.

Theorem 13 ([10]): The grand coalition is always the stable and sum-rate maximizing coalition for the receiver cooperation game using a MMSE detector.

Proof: For a coalition $\mathcal{S}$, the linear MMSE receiver minimizes both the noise and the interference for the links in $\mathcal{S}$ by applying the linear transformation $L_{\mathcal{S}} = \left[\mathbf{R}_{\mathcal{S}} + \sigma^2\mathbf{A}_{\mathcal{S}}^2\right]^{-1}$. It can be shown that the SINR $\gamma_k(\mathcal{S})$ of transmitter k belonging to the coalition $\mathcal{S}$, for all $k \in \mathcal{S}$, is [20]

$$\gamma_k(\mathcal{S}) = \frac{\left[(\mathbf{L}_{\mathcal{S}}\mathbf{R}_{\mathcal{S}})_{kk}\right]^2 h_k^2 P}{\begin{pmatrix} \sigma^2\left(\mathbf{L}_{\mathcal{S}}\mathbf{R}_{\mathcal{S}}\mathbf{L}_{\mathcal{S}}\right)_{kk} + \rho^2\left[(\mathbf{L}_{\mathcal{S}}\mathbf{e}_{\mathcal{S}})_k\right]^2 \sum_{j\notin\mathcal{S}} h_j^2 P \\ + \sum_{j\in\mathcal{S}, j\neq k}\left[(\mathbf{L}_{\mathcal{S}}\mathbf{R}_{\mathcal{S}})_{kj}\right]^2 h_j^2 P \end{pmatrix}} \tag{28}$$

where $\mathbf{e}_{\mathcal{S}}$ is a vector of length $|\mathcal{S}|$ with entries $e_k = 1$ for all k. The second and third terms in the denominator of (28) are the interference presented to link k from other links outside and within $\mathcal{S}$, respectively. From (28) the SINR, and hence, the rate achieved by every transmitter is maximized when all users are a part of the grand coalition. Thus, every transmitter would prefer to belong to the grand coalition where it is not subject to additional interference from non-cooperating transmitters, i.e., the grand coalition is both sum-rate maximizing and stable. ■

Theorem 14: The grand coalition is the stable and sum-rate maximizing coalition in the high SNR regime for the receiver cooperation game using a decorrelating detector.

Proof: The SINR $\eta_k(\mathcal{S})$ achieved at the decorrelating receiver by every transmitter k in the coalition $\mathcal{S}$ is [20]

$$\eta_k(\mathcal{S}) = \frac{h_k^2 P}{\left[\frac{\sigma^2}{1-\rho}\cdot\frac{1+\rho(|\mathcal{S}|-2)}{1+\rho(|\mathcal{S}|-1)} + \left[\frac{\rho}{1+\rho(|\mathcal{S}|-1)}\right]^2 \sum_{j\notin\mathcal{S}} h_j^2 P\right]} \tag{29}$$

where the first and second terms in the denominator of (29) are the interference due to other links within and outside the coalition $\mathcal{S}$, respectively. Recall that the core of a NTU game is the set of all payoff profiles for which there is no coalition $\mathcal{S} \subset \mathcal{K}$ that can achieve a payoff vector $\underline{R}_{\mathcal{S}} = (R_k)_{k\in\mathcal{S}}$ such that $R_k(\mathcal{S}) > R_k(\mathcal{K})$ for all $k \in \mathcal{S}$. From (29), we see that the payoff of any link k when it is a part of the grand coalition is

$$\eta_k(\mathcal{K}) = \frac{h_k^2 P}{\frac{\sigma^2}{1-\rho}\cdot\frac{1+\rho(K-2)}{1+\rho(K-1)}}. \tag{30}$$

Further, comparing (29) and (30), in the high SNR regime we have

$$\lim_{\sigma\to 0}\eta_k(\mathcal{S}) < \lim_{\sigma\to 0}\eta_k(\mathcal{K}). \tag{31}$$

Thus, in the high SNR regime, the grand coalition is stable as every link achieves its largest SINR, and hence, rate, when it is a part of the grand coalition and therefore has no incentive to defect. ■

V. Transmitter Cooperation

A. Transmitter Cooperation: Perfect Transmit Side-Information

The K receivers jointly decode their received signals, and thus, can be considered as a distributed K-antenna receiver.

For any coalition structure $(\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_N)$ where $2 \leq N \leq K$, the IC simplifies to a MIMO-MAC with per-antenna power constraints such that the transmitters in a coalition act as a single transmitter with multiple antennas (see Fig. 5 for $N = 2$). For the GC $(N = 1)$ the cooperative channel further simplifies to a MIMO point-to-point channel with per antenna power constraints. From (6), we write the $K \times 1$ vector of received signals at the K receivers, $\underline{Y}_{\mathcal{K}}$, as

$$\underline{Y}_{\mathcal{K}} = \sum\nolimits_{n=1}^{N} \mathbf{H}_{\mathcal{S}_n} \underline{X}_{\mathcal{S}_n} + \underline{Z}_{\mathcal{K}} \tag{32}$$

where $\mathbf{H}_{\mathcal{S}_n}$ is a $K \times |\mathcal{S}_n|$ channel gains matrix, $\underline{X}_{\mathcal{S}_n}$ is an input vector whose i^{th} entry is the signal transmitted by the i^{th} transmitter in the coalition $\mathcal{S}_n$, and $\underline{Z}_{\mathcal{K}}$ is the noise vector whose k^{th} entry Z_k is the noise at the k^{th} receiver. For the received signals in (32), we obtain the sum-rate achieved by the coalition $\mathcal{S}_n$ as the capacity of a $|\mathcal{S}_n| \times K$ MIMO channel [18] subject to worst case interference from the users not in $\mathcal{S}_n$. This is a mutual information game [4, Chap. 10, p. 263] and thus the sum-rate of a coalition is both maximized and minimized by Gaussian signaling at the users in $\mathcal{S}_n$ and $\mathcal{S}_n^c$, respectively, for all n. Further, the rate achieved by transmitters in a coalition can be arbitrarily apportioned between its users and thus the transmitter cooperation game is a TU game. We henceforth refer to this game as a *transmitter cooperation jamming game*.

We write $\mathbf{Q}_{\mathcal{A}} = E[X_{\mathcal{A}} X_{\mathcal{A}}^{\dagger}]$ to denote the covariance matrix of the users in $\mathcal{A}$ for all $\mathcal{A} \subseteq \mathcal{K}$ where † denotes the conjugate transpose of a matrix and I_K for the identity matrix of size K. For Gaussian signaling, the value $v(\mathcal{S})$ of a coalition $\mathcal{S}$ of transmitters is given as

$$\begin{aligned} v(\mathcal{S}) &= \min_{\mathbf{Q}_{\mathcal{S}^c}} \max_{\mathbf{Q}_{\mathcal{S}}} I(X_{\mathcal{S}}; Y_{\mathcal{K}}) && (33) \\ &= \min_{\mathbf{Q}_{\mathcal{S}^c}} \max_{\mathbf{Q}_{\mathcal{S}}} \left\{ \log \left(\frac{\left| \mathbf{I}_K + \mathbf{H}_{\mathcal{K}} \mathbf{Q}_{\mathcal{K}} \mathbf{H}_{\mathcal{K}}^{\dagger} \right|}{\left| \mathbf{I}_K + \mathbf{H}_{\mathcal{S}^c} \mathbf{Q}_{\mathcal{S}^c} \mathbf{H}_{\mathcal{S}^c}^{\dagger} \right|} \right) \right\} && (34) \end{aligned}$$

such that the diagonal entries of $\mathbf{Q}_{\mathcal{A}}$ for all $\mathcal{A}$ are constrained by (7) as

$$(\mathbf{Q}_{\mathcal{A}})_{kk} \leq P_k \quad \text{for all } k \in \mathcal{A}. \tag{35}$$

We use the following proposition on block diagonal matrix multiplication to further simplify (34).

Proposition 15: The product $\mathbf{AQA}^{\dagger}$ for a block diagonal matrix $\mathbf{Q}$ and $K \times K$ matrix $\mathbf{A}$ simplifies as

$$\mathbf{AQA}^{\dagger} = \mathbf{A}_{\mathcal{S}} \mathbf{Q}_{\mathcal{S}} \mathbf{A}_{\mathcal{S}}^{\dagger} + \mathbf{A}_{\mathcal{S}^c} \mathbf{Q}_{\mathcal{S}^c} \mathbf{A}_{\mathcal{S}^c}^{\dagger} \tag{36}$$

where $\mathbf{Q}_{\mathcal{S}}$ and $\mathbf{Q}_{\mathcal{S}^c}$ are square matrices and $\mathbf{A}_{\mathcal{S}}$ and $\mathbf{A}_{\mathcal{S}^c}$ are $K \times |\mathcal{S}|$ and $K \times |\mathcal{S}^c|$ matrices, respectively, such that

$$\mathbf{Q} = \begin{pmatrix} \mathbf{Q}_{\mathcal{S}} & 0 \\ 0 & \mathbf{Q}_{\mathcal{S}^c} \end{pmatrix} \quad \text{and} \quad \mathbf{A} = \begin{pmatrix} \mathbf{A}_{\mathcal{S}} & \mathbf{A}_{\mathcal{S}^c} \end{pmatrix}. \tag{37}$$

Proof: The proof follows simply from expanding $\mathbf{Q}$ and $\mathbf{A}$ as in (37), respectively, such that

$$\begin{aligned} \mathbf{AQA}^{\dagger} &= \begin{pmatrix} \mathbf{A}_{\mathcal{S}} & \mathbf{A}_{\mathcal{S}^c} \end{pmatrix} \begin{pmatrix} \mathbf{Q}_{\mathcal{S}} & 0 \\ 0 & \mathbf{Q}_{\mathcal{S}^c} \end{pmatrix} \begin{pmatrix} \mathbf{A}_{\mathcal{S}}^{\dagger} \\ \mathbf{A}_{\mathcal{S}^C}^{\dagger} \end{pmatrix} && (38) \\ &= \begin{pmatrix} \mathbf{A}_{\mathcal{S}} \mathbf{Q}_{\mathcal{S}} & \mathbf{A}_{\mathcal{S}^c} \mathbf{Q}_{\mathcal{S}^c} \end{pmatrix} \begin{pmatrix} \mathbf{A}_{\mathcal{S}}^{\dagger} \\ \mathbf{A}_{\mathcal{S}^C}^{\dagger} \end{pmatrix} && (39) \end{aligned}$$

which simplifies to (36). ■

Since the transmitted signals of users across competing coalitions $\mathcal{S}$ and $\mathcal{S}^c$ are independent, we use Proposition 15 to simplify the $\log$ expression in (34) as

$$v(\mathcal{S}) = \min_{\mathbf{Q}_{\mathcal{S}^c}} \max_{\mathbf{Q}_{\mathcal{S}}} \log \left(\frac{\left| \mathbf{I}_K + \mathbf{H}_{\mathcal{S}} \mathbf{Q}_{\mathcal{S}} \mathbf{H}_{\mathcal{S}}^{\dagger} + \mathbf{H}_{\mathcal{S}^c} \mathbf{Q}_{\mathcal{S}^c} \mathbf{H}_{\mathcal{S}^c}^{\dagger} \right|}{\left| \mathbf{I}_K + \mathbf{H}_{\mathcal{S}^c} \mathbf{Q}_{\mathcal{S}^c} \mathbf{H}_{\mathcal{S}^c}^{\dagger} \right|} \right). \tag{40}$$

To simplify the optimization in (40), we use the following two lemmas on functions of symmetric semi-definite matrices where we write $\mathbb{S}_{+}^{n}$ to denote the set of such matrices.

Lemma 16 ([21]): The function $f : \mathbb{S}_{+}^{n} \mapsto \mathbb{R}$ defined as

$$f(K_z) = \log\left(|K_x + K_z| / |K_z| \right) \tag{41}$$

is convex in K_z given K_x is symmetric positive semi-definite. The convexity is strict if K_x is positive definite.

Lemma 17 ([21]): The function $g : \mathbb{S}_{+}^{n} \mapsto \mathbb{R}$ defined as

$$g(K_x) = \log\left(|K_x + K_z| / |K_z| \right) \tag{42}$$

is strictly concave in K_x given K_z is symmetric positive definite.

We use the preceding Lemmas 16 and 17 to prove the saddle point property of the transmitter cooperation jamming game. For ease of exposition, we henceforth write $l(\mathbf{Q}_{\mathcal{S}}, \mathbf{Q}_{\mathcal{S}^c})$ to denote the $\log$ expression in (40).

Lemma 18: The transmitter cooperation jamming game has a saddle point solution such that

$$l(\mathbf{Q}_{\mathcal{S}}, \mathbf{Q}_{\mathcal{S}^c}^{*}) \leq l(\mathbf{Q}_{\mathcal{S}}^{*}, \mathbf{Q}_{\mathcal{S}^c}^{*}) \leq l(\mathbf{Q}_{\mathcal{S}}^{*}, \mathbf{Q}_{\mathcal{S}^c}) \tag{43}$$

and

$$\max_{\mathbf{Q}_{\mathcal{S}}} \min_{\mathbf{Q}_{\mathcal{S}^c}} l(\mathbf{Q}_{\mathcal{S}}, \mathbf{Q}_{\mathcal{S}^c}) = \min_{\mathbf{Q}_{\mathcal{S}^c}} \max_{\mathbf{Q}_{\mathcal{S}}} l(\mathbf{Q}_{\mathcal{S}}, \mathbf{Q}_{\mathcal{S}^c}) \tag{44}$$

where $\mathbf{Q}_{\mathcal{S}}^{*}$ and $\mathbf{Q}_{\mathcal{S}^c}^{*}$ are covariance matrices that maximize and minimize $l(\mathbf{Q}_{\mathcal{S}}, \mathbf{Q}_{\mathcal{S}^c})$ in (40), respectively.

Proof: The proof follows from the fact that the transmitter cooperation jamming game is a mutual information game (see [4, Chap. 10, p. 263]). Further from Lemmas 16 and 17, the game has a saddle point at $(\mathbf{Q}_{\mathcal{S}}^{*}, \mathbf{Q}_{\mathcal{S}^c}^{*})$ satisfying (43) such that a deviation from the optimal matrix for either $\mathcal{S}$ or $\mathcal{S}^c$ worsens $l(\mathbf{Q}_{\mathcal{S}}, \mathbf{Q}_{\mathcal{S}^c})$ from that coalition's standpoint [4, Chap. 10, p. 263]. ■

Theorem 19: The transmitter cooperation jamming game is cohesive.

Proof: From Definition 2 and Remark 3, the game is cohesive when

$$v(\mathcal{K}) \geq \sum\nolimits_{i=1}^{N} v(\mathcal{S}_i) \tag{45}$$

where $\mathcal{S}_1, \ldots, \mathcal{S}_N$ is any partition of $\mathcal{K}$, and the value $v(\mathcal{S}_i)$ of coalition $\mathcal{S}_i$ is obtained from (40) by setting $\mathcal{S} = \mathcal{S}_i$. The value $v(\mathcal{K})$ of the GC is given by (40) with $\mathcal{S} = \mathcal{K}$ and $\mathcal{S}^c = \emptyset$. Consider a coalition structure $\mathcal{S}_1, \ldots, \mathcal{S}_N$, for any $1 < N \leq K$. We expand $I(X_{\mathcal{K}}; Y_{\mathcal{K}})$ as

$$\begin{aligned} I(X_{\mathcal{K}}; Y_{\mathcal{K}}) &= I(X_{\mathcal{S}_1}, \ldots, X_{\mathcal{S}_N}; Y_{\mathcal{K}}) && (46) \\ &\geq \sum\nolimits_{i=1}^{N} I(\mathbf{X}_{\mathcal{S}_i}; \mathbf{Y}_{\mathcal{K}}) && (47) \end{aligned}$$

where the inequality in (47) follows from chain rule of mutual information [4, Theorem 2.5.2] and the fact that conditioning does not increase entropy. Consider the block diagonal matrix $\mathbf{Q}_{\mathcal{K}}^{(bd)}$

$$\mathbf{Q}_{\mathcal{K}}^{(bd)} = \begin{pmatrix} \mathbf{Q}_{\mathcal{S}_1}^* & 0 & 0 & \dots \\ 0 & \mathbf{Q}_{\mathcal{S}_2}^* & 0 & \dots \\ 0 & 0 & \ddots & \dots \\ \vdots & \vdots & \dots & \mathbf{Q}_{\mathcal{S}_N}^* \end{pmatrix} \quad (48)$$

where $\mathbf{Q}_{\mathcal{S}_i}^*$ is the maximizing covariance matrix for $v(\mathcal{S}_i)$ for all i and all partitions. From (48), the covariance matrix $\mathbf{Q}_{\mathcal{S}_i^c}$ of the users in $\mathcal{S}_i^c$ is obtained from $\mathbf{Q}_{\mathcal{K}}^{(bd)}$ by deleting the rows and columns corresponding to users in $\mathcal{S}_i$. In the following inequalities we write $(\cdot)_{\mathbf{Q}_{\mathcal{K}}^*}$ to denote that the expression $(\cdot)$ is evaluated at $\mathbf{Q}_{\mathcal{K}}^*$. We lower bound $v(\mathcal{K})$ as

$$v(\mathcal{K}) = [I(\mathbf{X}_{\mathcal{K}};\mathbf{Y}_{\mathcal{K}})]_{\mathbf{Q}_{\mathcal{K}}^*} \geq [I(\mathbf{X}_{\mathcal{K}};\mathbf{Y}_{\mathcal{K}})]_{\mathbf{Q}_{\mathcal{K}}^{(bd)}} \quad (49)$$

$$\geq \left[\sum_{i=1}^{N} I(\mathbf{X}_{\mathcal{S}_i};\mathbf{Y}_{\mathcal{K}})\right]_{\mathbf{Q}_{\mathcal{K}}^{(bd)}} \quad (50)$$

$$= \sum_{i=1}^{N} \log\left\{\frac{\left|\mathbf{I}+\mathbf{H}_{\mathcal{S}_i}\mathbf{Q}_{\mathcal{S}_i}^*\mathbf{H}_{\mathcal{S}_i}^\dagger+\mathbf{H}_{\mathcal{S}_i^c}\mathbf{Q}_{\mathcal{S}_i^c}\mathbf{H}_{\mathcal{S}_i^c}^\dagger\right|}{\left|\mathbf{I}+\mathbf{H}_{\mathcal{S}_i^c}\mathbf{Q}_{\mathcal{S}_i^c}\mathbf{H}_{\mathcal{S}_i^c}^\dagger\right|}\right\} \quad (51)$$

$$\geq \sum_{i=1}^{N} \log\left\{\frac{\left|\mathbf{I}+\mathbf{H}_{\mathcal{S}_i}\mathbf{Q}_{\mathcal{S}_i}^*\mathbf{H}_{\mathcal{S}_i}^\dagger+\mathbf{H}_{\mathcal{S}_i^c}\mathbf{Q}_{\mathcal{S}_i^c}^*\mathbf{H}_{\mathcal{S}_i^c}^\dagger\right|}{\left|\mathbf{I}+\mathbf{H}_{\mathcal{S}_i^c}\mathbf{Q}_{\mathcal{S}_i^c}^*\mathbf{H}_{\mathcal{S}_i^c}^\dagger\right|}\right\} \quad (52)$$

$$= \sum_{i=1}^{N} v(\mathcal{S}_i) \quad (53)$$

where (49) follows from Lemmas 17 and 18, (50) follow from (47), (51) follows from Proposition 15 and evaluating the resulting expression at $\mathbf{Q}_{\mathcal{K}}^{(bd)}$, (52) follows from Lemma 18, and (53) follows from (40). Note that the $\mathbf{Q}_{\mathcal{S}_i^c}^*$ in (52) is the minimizing matrix in (40) for $\mathcal{S} = \mathcal{S}_i$. ■

For cohesive games [8, p. 258], the grand coalition is the only possible stable coalition structure. To determine the stability of the GC for the transmitter cooperation jamming game, i.e., to verify whether the core of this game is non-empty, we need to show that the GC is guaranteed to have at least one stable payoff profile. An analytical proof for the core is intractable since it requires comparing K-dimensional rate regions that are functions of the channel and power parameters. Instead, using the simple linear programming interpretation described in Section II, we present a numerical example that illustrates that the core can be empty.

Example 20: Consider a 3-link IC with perfectly cooperating receivers. All the transmitters have a maximum power constraint of unity and the channel matrix $\mathbf{H}_{\mathcal{K}}$ with entries $h_{m,k}$ between the m^{th} receiver and k^{th} transmitter is

$$\mathbf{H} = \begin{pmatrix} 0.3019 & 0.3772 & 1.8021\times10^{-2} \\ 2.6256\times10^{-8} & 3.1413\times10^{-5} & 2.5662\times10^{-5} \\ 2.6893\times10^{-6} & 1.9941\times10^{-3} & 0.8502 \end{pmatrix}. \quad (54)$$

From (3) and (4) in Section II, for the $\mathbf{H}$ in (54), the existence of a core with non-zero rate tuples $(R_1, R_2, \dots, R_K)$ is equivalent to the feasibility of the linear program given by $\sum_{k\in\mathcal{S}} R_k \geq v(\mathcal{S})$ for all $\mathcal{S} \subseteq \mathcal{K}$ where $v(\mathcal{S})$ is defined as in (40). Numerical evaluation reveals that there does not exist a feasible rate vector where all users achieve rates larger than what they can achieve outside the GC, i.e., the core is empty. As a result the GC is not stable since a subset of users that can achieve better rates as a coalition will break away. Note however, that no other coalition structure is stable either. This is because users breaking away can be incentivized with larger payoffs by those users who do not wish to leave the GC. This in turn will result in a different subset of users attempting to leave the GC for better rates and thus, the game results in an oscillatory behavior instead of a single convergent stable structure (see also [8, p. 259]). Finally, our numerical analyses lead us to conjecture that the core will be non-empty, i.e., the GC will be stable, when the channel gains $h_{m,k}$ as well as the powers P_k for all m and k are comparable (see [22, Chap. 4] for details).

Remark 21: The stability of the grand coalition is equivalent to verifying the feasibility of the linear program given by (3) and (4). Furthermore, (3) and (4) also determine the set of conditions on the channel gains and transmit powers required to achieve a non-empty core.

B. Transmitter Cooperation: Partial Decode-and-Forward (PDF)

We now seek to understand if relaxing the assumption of perfect noiseless links between the transmitters can still result in the GC as the only candidate for the core. We thus consider a clustered model introduced in equation (12) where the full-duplex transmitters have noisy inter-user channels and the receivers are co-located. For this model, we consider a PDF strategy, introduced in [13, Chap. 7] for a two-user cooperative MAC, and later extended in [14] for $K > 2$.

Consider a coalition $\mathcal{S} \subseteq \mathcal{K}$ of users that cooperate. In the PDF strategy, user $k \in \mathcal{S}$ transmits the two new messages $w_{k,1} \in \{1, 2, \dots, 2^{nR_{k,1}}\}$ and $w_{k,2} \in \{1, 2, \dots, 2^{nR_{k,2}}\}$ and a cooperative message $w_0 \in \{1, 2, \dots, 2^{nR_0}\}$ where $R_{k,1}$, $R_{k,2}$, and R_0 are the rates in bits per channel use at which the messages $w_{k,1}$, $w_{k,2}$, and w_0 are transmitted, respectively, and n is the number of channel uses [14]. The signal X_k transmitted by user k is

$$X_k = X_{k,d} + V_{k,c} + U \quad \text{for all } k \in \mathcal{S} \quad (55)$$

where $X_{k,d}$, $V_{k,c}$, and U are zero-mean independent Gaussian random variables that carry the messages $w_{k,1}$, $w_{k,2}$, and w_0 and have variances $p_{k,d}$, $p_{k,c}$, and $p_{k,u}$, respectively, such that the total power p_k at user k subject to (7) is

$$p_k = p_{k,d} + p_{k,c} + p_{k,u} \leq P_k \quad \text{for all } k \in \mathcal{S}. \quad (56)$$

The stream $w_{k,2}$ is decoded by all cooperating users while the destination decodes all streams.

As with previous analysis for perfectly cooperating transmitters, in evaluating the value of a coalition we assume that the users outside a coalition cooperate to act as worst case jammers and transmit Gaussian signals that are independent of the signals of the users in the coalition. We show that the

PDF jamming game is an NTU game. To this end, we first determine the PDF rate region by applying the result in [14, Thrm. 1]. Let $\mathcal{G} \subseteq \mathcal{S}$ and $\mathcal{G}^c$ be the complement of $\mathcal{G}$ in $\mathcal{S}$. We write $R_{\mathcal{G},j} = \sum_{m\in\mathcal{G}} R_{m,j}$, $j = 1, 2$, $R_{\mathcal{G}} = R_{\mathcal{G},1} + R_{\mathcal{G},2}$, and the cardinality of $\mathcal{G}$ as $|\mathcal{G}|$.

Theorem 22: For the PDF jamming game, a rate tuple for a coalition $\mathcal{S}$ is achievable if, for all $\mathcal{G} \subseteq \mathcal{S}$, it satisfies

$$R_{\mathcal{G},2} \leq \min_{m\in\mathcal{G}^c} \{I(V_{\mathcal{G}}; Y_m|X_m, U, V_{\mathcal{G}^c})\} \tag{57}$$

$$R_{\mathcal{G},1} \leq I(X_{\mathcal{G}}; Y_d|X_{\mathcal{G}^c}, V_{\mathcal{S}}, U) \tag{58}$$

$$R_{\mathcal{S}} \leq I(X_{\mathcal{S}}; Y_d). \tag{59}$$

Proof: The proof follows directly from [14, Thrm. 1] assuming worst case jamming from users outside $\mathcal{S}$. ■

A bound on the sum-rate $R_{\mathcal{G},2}$, for all $\mathcal{G} \subset \mathcal{S}$, results from jointly decoding the messages $w_{k,2}$, for all $k \in \mathcal{G}$, at a cooperating user $m \notin \mathcal{G}$. We obtain the bound in (57) by taking the smallest bound over all such $m \in \mathcal{S}$. The bound in (58) results from decoding $w_{k,1}$, for all $k \in \mathcal{G}$, at the destination. Finally, the bound in (59) results from decoding all messages at the destination. We obtain the bounds on $R_{\mathcal{G}}$, for all $\mathcal{G} \subseteq \mathcal{S}$, by summing the bounds on $R_{\mathcal{G},1}$ and $R_{\mathcal{G},2}$. The bounds on $R_{\mathcal{G},1}$ are given by (58). We henceforth denote this bound as $B_{\mathcal{G},1}$. On the other hand, in addition to the bound in (57), for any partition $(\mathcal{G}_1, \mathcal{G}_2, \ldots, \mathcal{G}_N)$ of $\mathcal{G}$ such that $1 \leq N \leq |\mathcal{G}|$, a bound on $R_{\mathcal{G},2}$ is obtained as a sum of the bounds on $R_{\mathcal{G}_n}$, i.e., from the fact that $R_{\mathcal{G},2} = \sum_{n=1}^{N} R_{\mathcal{G}_n,2}$. Thus, the smallest bound on $R_{\mathcal{G},2}$ is a minimum over all such partitions. Let $(\mathcal{G}_1^*, \mathcal{G}_2^*, \ldots, \mathcal{G}_N^*)$ be the minimizing partition. Further, from (57), we see that for each $\mathcal{G}_n^*$, there exists an index m_n^* denoting the decoding user at which the bound on $R_{\mathcal{G}_n^*,2}$ is a minimum. We write this smallest bound on $R_{\mathcal{G},2}$ as $B_{\mathcal{G},2}(\{\mathcal{G}_n^*, m_n^*\}_N)$ to denote the dependence of the bound on the minimizing partition and indexes such that $R_{\mathcal{G}} \leq B_{\mathcal{G},1} + B_{\mathcal{G},2}(\{\mathcal{G}_n^*, m_n^*\}_N)$. We obtain an achievable rate region for the users in a coalition $\mathcal{S}$ by substituting (55) in (57)-(59) for each choice of $(p_{k,d}, p_{k,c}, p_{k,u})$ subject to (56) and for all $k \in \mathcal{S}$. We write $\underline{P}$ to denote the vector of tuples $(p_{k,d}, p_{k,c}, p_{k,u})$ for all $k \in \mathcal{S}$ and $\mathcal{R}_{\mathcal{S}}(\underline{P})$ for the rate region achieved for each choice of $\underline{P}$. For this signaling, the bounds in (58) are concave functions of $p_{k,d}$ while that in (59) depend only p_k and $p_{k,u}$ for all $k \in \mathcal{G}$. However, the bounds in (57) are not concave functions since they include interference from $p_{k,d}$ for all $k \neq m$. Thus, the PDF rate region $\mathcal{R}_{\mathcal{S}}^{PDF}$ is obtained as

$$\mathcal{R}_{\mathcal{S}}^{PDF} = co\left(\bigcup\nolimits_{\underline{P}} \mathcal{R}_{\mathcal{S}}(\underline{P})\right) \tag{60}$$

where co denotes the convex hull operation. Further, each rate tuple on the hull may be achieved by a different $\underline{P}$. We define the value, $\mathcal{V}(\mathcal{S})$, of a coalition $\mathcal{S}$ as a K-dimensional rate region where the rates achieved by the users in $\mathcal{S}$ belongs to the largest achievable $\mathcal{R}_{\mathcal{S}}^{PDF}$ while those for the users not in $\mathcal{S}$ can take arbitrary values in the $|\mathcal{S}^c|$-dimensional orthant $\mathbb{R}_+^{|\mathcal{S}^c|}$. For this $\mathcal{V}(\mathcal{S})$, from Definition 8, the PDF jamming game is an NTU game.

To determine the core of this game, one has to verify if the game is cohesive, i.e., if the GC rate region, $\mathcal{V}(\mathcal{K})$, satisfies (5) for all partitions $(\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_N)$, $2 \leq N \leq K$. We begin by determining the power allocations that maximize the region $\mathcal{R}_{\mathcal{S}}^{PDF}$ for any coalition. Maximizing $\mathcal{R}_{\mathcal{S}}^{PDF}$ is not a straightforward optimization problem since the rate bounds in (57) are not in general concave functions of $\underline{P}$. To alleviate this problem, we will build on the result in [15, Proposition 1] where it has been shown that for a two-user cooperative MAC, irrespective of the channel gains, the power allocation that maximizes the rate region simplifies to setting either $p_{k,d} = 0$ or $p_{k,c} = 0$ for all k and for all choices of p_k and $p_{k,u}$ subject to (56). This allocation also simplifies the rate bounds to concave functions of power that can be maximized using convex optimization techniques. We prove a similar result for the clustered model and for arbitrary K. Our result has the intuitive interpretation that the clustered users benefit from exploiting their strong inter-user gains to decode and forward all messages for each other, i.e., in addition to w_0, each user transmits only one message stream which is decoded by all other cooperating users.

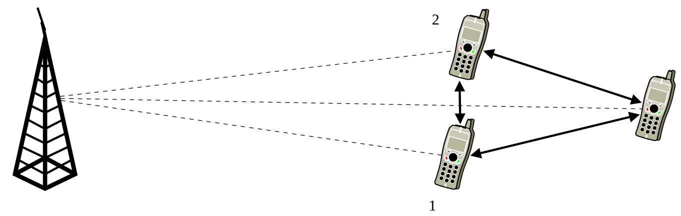

Fig. 6. A three-user clustered MAC.

Theorem 23: The rate region $R_{\mathcal{S}}^{PDF}$ of a coalition $\mathcal{S}$ of clustered users, for all $\mathcal{S} \subseteq \mathcal{K}$, is maximized when user k sets $p_{k,d} = 0$, for all $k \in \mathcal{S}$.

Proof: We assume Gaussian signaling for all the users in $\mathcal{K}$. For the users in $\mathcal{S}$ we choose the signals as in (55) and fix the transmit and cooperative powers p_k and $p_{k,u}$ such that the remaining power $\tilde{p}_k \triangleq p_k - p_{k,u}$ is split between $p_{k,d}$ and $p_{k,c}$ for all $k \in \mathcal{S}$. The region $\mathcal{R}_{\mathcal{S}}^{PDF}$ is given by (60) for all choices of $(p_k, p_{k,c}, p_{k,u})$. We develop the results for the $|\mathcal{S}|$-user sum-rate bound $R_{\mathcal{S}}$. One can extend the proof in a straightforward manner to the bounds on $R_{\mathcal{G}}$ for any $\mathcal{G} \subset \mathcal{S}$. Without loss of generality, we write the jamming noise seen by a coalition $\mathcal{S}$ as $(J_{\mathcal{S}} - 1)$ and scale the signal powers in (56) for all $k \in \mathcal{S}$ by $J_{\mathcal{S}}$ such the total interference and noise power is unity. Since the bound in (59) is independent of $p_{k,d}$ and $p_{k,c}$ for a fixed p_k and $p_{k,u}$, we focus on the bounds on $R_{\mathcal{S}}$ obtained as a sum of the bounds on $R_{\mathcal{S},1}$ and $R_{\mathcal{S},2}$. Let $\{\mathcal{S}_n^*, m_n^*\}_N$ be a partition of $\mathcal{S}$ and a collection of indexes that jointly achieve the smallest bound on $R_{\mathcal{S},2}$ such that

$$R_{\mathcal{S}} \leq B_{\mathcal{S},1} + B_{\mathcal{S},2}(\{S_n^*, m_n^*\}_N) \tag{61}$$

where as described earlier $B_{\mathcal{S},1}$ and $B_{\mathcal{S},2}$ are the smallest bounds on $R_{\mathcal{S},1}$ and $R_{\mathcal{S},2}$, respectively. For the Gaussian signaling in (55), using (10) and (11) these terms simplify as

$$B_{\mathcal{S},1} = \log\left(1 + \sum\nolimits_{i\in\mathcal{S}} h_{d,i} p_{i,d}\right) \tag{62}$$

$$B_{\mathcal{S},2}\left(\{S_n^*, m_n^*\}_N\right) = \log \prod_{n=1}^{N} \left(1 + \frac{\sum\limits_{i\in\mathcal{S}_n^*} h_{m_n^*,i} p_{i,c}}{1 + \sum\limits_{j\in\mathcal{S}, j\neq m_n^*} h_{m_n^*,j} p_{j,d}}\right). \tag{63}$$

Observe that $B_{\mathcal{S},1}$ in (62) is an increasing function of $p_{i,d}$ while $B_{\mathcal{S},2}$ in (61) and (63) is decreasing in $p_{i,d}$, for all $i \in \mathcal{S}$. Therefore, it is not immediately clear whether setting $p_{i,d} = 0$ for all $i \in \mathcal{S}$ would maximize the bounds in (61). Consider the case where $p_{i,d} = 0$, such that $p_{i,c} = \tilde{p}_i$, for all $i \in \mathcal{S}$. Denoting the minimizing partitions and indexes for this case by S_n and m_n, respectively, for all $n = 1, 2, \ldots, N$, the bounds in (61) simplify as

$$R_{\mathcal{S}}|_{p_{i,d}=0} \leq B_{\mathcal{S},2}\left(\{S_n, m_n\}_N\right)|_{p_{i,d}=0}. \tag{64}$$

On the other hand, for any $p_{i,d} > 0$, we denote the minimizing partitions and indexes by $\mathcal{S}_t'$ and m_t' where $t = 1, \ldots, T$, and rewrite (61) as

$$R_{\mathcal{S}}|_{p_{i,d}>0} \leq B_{\mathcal{S},1} + B_{\mathcal{S},2}\left(\{S_t', m_t'\}_T\right). \tag{65}$$

Using the identity $(1 + \sum_k x_k) \leq \Pi_k (1 + x_k)$, for all $x_k > 0$, we upper bound $B_{\mathcal{S},1}$, and thus, $R_{\mathcal{S}}|_{p_{i,d}>0}$ in (65) with

$$\begin{aligned} &\log\left[\left(1 + \sum_{i\in\mathcal{S}_1} h_{d,i} p_{i,d}\right) \ldots \left(1 + \sum_{i\in\mathcal{S}_N} h_{d,i} p_{i,d}\right)\right] \quad (66)\\ &\quad + B_{\mathcal{S},2}\left(\{S_t', m_t'\}_T\right) \\ &\leq \log\left[\left(1 + \sum_{i\in\mathcal{S}_1} h_{d,i} p_{i,d}\right) \ldots \left(1 + \sum_{i\in\mathcal{S}_N} h_{d,i} p_{i,d}\right)\right] \quad (67)\\ &\quad + B_{\mathcal{S},2}\left(\{S_n, m_n\}_N\right) \end{aligned}$$

where the inequality in (67) follows from the fact that for the chosen values of $p_{i,d} > 0$, for all $i \in \mathcal{S}$, the set $\{S_t', m_t'\}_T$ results in the smallest bound on $R_{\mathcal{S},2}$. To show that the bound in (65) is smaller than that in (64), from (66) and (67), it suffices to show that $B_{\mathcal{S},2}\left(\{S_n, m_n\}_N\right)|_{p_{i,d}=0}$ is upper bounded by

$$\log\left[\left(1 + \sum_{i\in\mathcal{S}_1} h_{d,i} p_{i,d}\right) \ldots \left(1 + \sum_{i\in\mathcal{S}_N} h_{d,i} p_{i,d}\right)\right] + B_{\mathcal{S},2}\left(\{S_n, m_n\}_N\right). \tag{68}$$

Expanding (68) using (62) and (63) and rearranging the terms, we need to show that

$$\prod_{n=1}^{N}\left[\frac{\left(1 + \sum_{i\in\mathcal{S}_n} h_{m_n,i}\tilde{p}_i\right)}{\left(1 + \frac{\sum_{i\in\mathcal{S}_n} h_{m_n,i} p_{i,c}}{1 + \sum_{j\neq m_n} h_{m_n,j} p_{j,d}}\right)}\right] \geq \prod_{n=1}^{N}\left(1 + \sum_{i\in\mathcal{S}_n} h_{d,i} p_{i,d}\right). \tag{69}$$

Simplifying (69) further, it suffices to show that, for all $n = 1, 2, \ldots, N$,

$$\left(1 + \frac{\sum_{i\in\mathcal{S}_n} h_{m_n,i} p_{i,c}}{1 + \sum_{j\neq m_n} h_{m_n,j} p_{j,d}}\right) \leq \frac{\left(1 + \sum_{i\in\mathcal{S}_n} h_{m_n,i}\tilde{p}_i\right)}{\left(1 + \sum_{i\in\mathcal{S}_n} h_{d,i} p_{i,d}\right)}. \tag{70}$$

Recall that $\tilde{p}_i$ and $p_{i,c}$ are the powers for transmitting $w_{k,c}$ when $p_{i,d} = 0$ and $p_{i,d} \neq 0$, respectively. For a fixed p_i and $p_{i,u}$, since $\tilde{p}_i > p_{i,c}$ we can expand $h_{m_n,i}\tilde{p}_i$ as $h_{m_n,i} p_{i,c} + h_{d,i} p_{i,d} + (h_{m_n,i} - h_{d,i}) p_{i,d}$, for all $i \in S_n$. We also expand

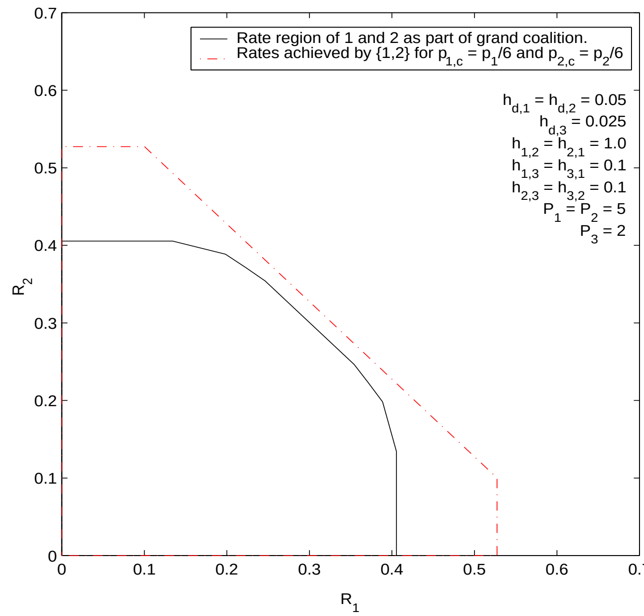

Fig. 7. Rate regions for the GC and the $\{1, 2\}$ coalition in the R_1-R_2 plane.

the denominator of the term to the left side of the inequality in (70) over all $j \in \mathcal{S}$ with $j \neq m_n$, where $m_n \in \mathcal{S}_n^c = \mathcal{S} \backslash \mathcal{S}_n$. With these two expansions (70) simplifies to requiring

$$\frac{\sum_{i\in\mathcal{S}_n} h_{m_n,i} p_{i,c}}{1 + \sum_{j\in\mathcal{S}_n} h_{m_n,j} p_{j,d} + \sum_{j\in\mathcal{S}_n^c, j\neq m_n} h_{m_n,j} p_{j,d}} \leq \frac{\left(\sum_{i\in\mathcal{S}_n} h_{m_n,i} p_{i,c} + \sum_{i\in\mathcal{S}_n} (h_{m_n,i} - h_{d,i}) p_{i,d}\right)}{\left(1 + \sum_{i\in\mathcal{S}_n} h_{d,i} p_{i,d}\right)}. \tag{71}$$

Comparing the numerators and denominators on both sides of (71), for the clustered model where $h_{m_n,i} > h_{d,i}$ for all i, one can easily see that the inequality is satisfied. Thus, for any $(p_i, p_{i,u})$, setting $p_{i,d} = 0$, for all i, maximizes the bound on $R_{\mathcal{S}}$. One can similarly show that the rate bounds for all $\mathcal{G} \subset \mathcal{S}$ are also maximized, and thus, the region $\mathcal{R}_{\mathcal{S}}\left(\underline{P}\right)$ is maximized. Since the argument holds for all $\underline{P}$ the rate tuples on the hull of $\mathcal{R}_{\mathcal{S}}^{PDF}$ are also maximized. ■

Thus, from Theorem 23, setting $p_{k,d} = 0$ for all k simplifies the bounds in (61)-(63) to concave functions of P_k for all $k \in \mathcal{S}$. As a result, the rate region $\mathcal{R}_{\mathcal{S}}^{PDF}$ does not require the convex hull operation in (60) thus simplifying the evaluation of $\mathcal{V}(S)$ for any $\mathcal{S}$. From Definitions 8 and 9, a necessary condition for the game to be cohesive is that, for every $\mathcal{S} \subset \mathcal{K}$, the projection of $\mathcal{V}(\mathcal{S})$ in the rate space of $\mathcal{S}$, i.e. $\mathcal{R}_{\mathcal{S}}^{PDF}$, is a subset of the projection to the same space of the GC value set, $\mathcal{V}(\mathcal{K})$. While Theorem 23 allows computing $\mathcal{V}(S)$ relatively easily, in general, inferences on the cohesiveness of the game cannot be drawn easily for arbitrary values of channel gains, user powers, and for any K. We thus use an example to illustrate that the PDF user cooperation game may not be cohesive, i.e., the GC may not achieve the largest rate region. In fact, our example reveals that for asymmetric inter-user channel gains and a few weak jammers, users can form smaller coalitions to achieve larger rates relative to the GC.

Remark 24: Verifying whether the grand coalition is cohesive is equivalent to verifying whether the conditions in (5) hold, i.e., (5) captures the functional dependence between the channel parameters and transmit powers required for the NTU game to be cohesive. In general, however, verifying the requirement that the intersection of the K-dimensional rate regions corresponding to all possible coalition structures lies within the GC rate region in (5) is not straightforward. Furthermore, the verification complexity grows exponentially in K.

Example 25: Consider a cooperative MAC shown in Fig. 6 with 3 users labeled $1, 2$, and 3 that are clustered as in (12) with gains $h_{d,1} = h_{d,2} = 0.05$, $h_{d,3} = 0.025$, $h_{1,2} = h_{2,1} = 1$, $h_{1,3} = h_{3,1} = h_{2,3} = h_{3,2} = 0.1$, and power constraints $P_1 = P_2 = 5$, and $P_3 = 2$. Thus, users 1 and 2 have a stronger inter-user channel to each other than to user 3 while user 3 has a smaller transmit power. In Fig. 7, we plot the rate region achieved by 1 and 2 when they are part of the grand coalition, i.e., we plot the projection of the GC region $\mathcal{V}(\{1, 2, 3\})$ on the R_1-R_2 plane computed using the bounds in (57)-(58) and Theorem 23. Also shown is the rate region achieved by users 1 and 2 as a coalition $\{1, 2\}$ from Theorem 23 for $p_{1,c} = p_{2,c} = P_1/6$ and assuming maximum jamming by user 3. Since the latter region contains the former, the game is not cohesive. Further, for every rate tuple achieved by users 1 and 2 when they are a part of the GC, there exists at least a tuple where both users achieve larger rates for the coalition $\{1, 2\}$, and thus, the GC is not stable. This is because the requirement of decoding the messages from users 1 and 2 at the relatively distant user 3 for the GC results in tighter bounds than those achieved by the coalition $\{1, 2\}$ in the presence of a weak jammer 3.

VI. Concluding Remarks

We have studied the stability of the GC when users in a wireless network are allowed to cooperate while maximizing their own rates. For an IC, we have shown that when only receivers are allowed to cooperate by jointly decoding their received signals, the GC is both stable and sum-rate optimal. However, we have shown that if the receivers cooperated using linear multiuser detectors, they cannot arbitrarily share the gains from cooperation and the stability of the GC depends on the SNR regime and the detector. We have also studied transmitter cooperation in an IC with perfectly cooperating receivers. We have shown that when transmitters are allowed to cooperate via noise-free links the GC is sum-rate optimal but may not be stable. Finally, we have shown that for a network where clustered transmitters cooperate by mutually decoding messages via PDF, the optimality of the GC from both a stability and a rate region perspective depends on the network geometry and the jamming potential of the users. For transmitter cooperation, we have presented a jamming interpretation to characterize the value of a coalition. Although the assumption represents an extreme adversarial response of the complementary coalition, it lower bounds the rates achieved by a coalition that breaks away from the GC, and is therefore a strong result when the core is empty. Our work has also demonstrated that stability depends on the incentives and disincentives that users have to cooperate. For example, the noise enhancement in a decorrelating detector can act as a disincentive to the stability of the GC in the low SNR regime. Similarly, channel gains and weak jammers can destabilize the GC when transmitters cooperate perfectly. Furthermore, noisy inter-user channels can also affect the stability of the GC for decoding transmitters.

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